Materials discussed on 27/9

We have discussed the following.

1. (a) Suppose u is harmonic function on $\mathbb{C}$, then whenever $B(p,r)$ is inside the domain of u,
$$u(p) = \fint_{B(p,r)} u(x)\, dx.$$
Hints: $u = Re(f)$ for some analytic function f, then use the cauchy integral formula.

   (b) And hence we have strong maximum principle: If u is harmonic function such that $u(x_0) = \sup_\Omega u$ for some interior point x_0, then u is constant.

2. Show Jensen formula by using the mean value property for harmonic functions.

   Hints: By dividing the conformal change of zeros, we may assume $f \neq 0$, hence we may apply mean value theorem on harmonic function $\log|f|$.

3. (a) We show the Schwarz lemma. (see textbook)

   (b) (Borel Caratheodory theorem) Suppose f is analytic on $B(R)$. Then for $r \in (0, R)$, we have the following inequality relating real part of f with f.
$$\sup_{B(r)} |f| \leq \frac{2r}{R-r} \sup_{B(R)} Re(f) + \frac{R+r}{R-r}|f(0)|.$$

   (c) We give a alternative way to show Lemma 5.5 in Ch5 by the mean of the above theorem.